

## Econometric Modelling of Financial Time Series

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### Abstract

This paper examines the relationship between assets, capital, liabilities, and liquidity in South Africa using the Johansen cointegration analysis and the GARCH model using times data for the period 02/2005 to 06/2018. The results obtained from the study suggest that the time series are integrated with order one,  $I(1)$ . The findings from the Johansen cointegration test indicated that the variables have a long-run cointegrating relationship. Furthermore, the results from the GARCH model revealed that the estimated model has statistically significant coefficients at 5% significance level. Additionally, results revealed that assets have a positive relationship with capital, liabilities, and liquidity. This implies that a percentage increase in assets will result in a percentage increase in capital, liabilities, and liquidity. The study found that assets, capital, liabilities, and liquidity are cointegrated for the financial-economic period of 2005/02 to 2018/06. The results also revealed that shocks decay quickly in the future and that the conditional variance is explosive. The diagnostic tests revealed that the estimated models show the characteristics of a well-specified model. The recommendations for future studies were formulated.

**Keywords:** *ARCH model; Cointegration; Financial time series; GARCH model; VECM; Volatility*



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### INTRODUCTION

The paper applies the cointegration technique and the generalized autoregressive conditional heteroscedastic (GARCH) to model financial time series data in the South African economy. Forecasting volatility can be helpful in the financial decision-making processes. This is especially applied to risk management and monetary policy process. The need for an accurate and appropriate volatility model for capturing conditional variance, forecasting, and estimation of financial data is becoming vital as many emerging economies become increasingly complex. One of the important areas of statistics is trying to understand how different variables react to each other using a given statistical technique. Apart from the mean, variables also react to one another through second moments. This means that the change in a given variable may result in a change in the measure of another variable which may also affect its volatility (De Wet, 2005). De Wet (2005) further suggested that to better compile a long-run strategy, the cointegration analysis is the most suited technique when it comes to assessing the long-run relationships among variables. Understanding the trends, characteristics, and relationships between variables is important in the current study.

The cointegration techniques and the GARCH models have been commonly utilized by academics and researchers since their introduction in the 1980s. The Johansen cointegration technique developed by Johansen and Juselius (1990) aimed to examine whether non-stationary time series data are integrated into the long run. The Johansen cointegration technique provides a means to examine whether a selected number of endogenous variables for an emerging economy share a joint long-run stochastic trend while allowing for feasible short-run divergences (Bagchi *et al.*, 2016).

According to Francq and Zakoian (2010), GARCH models prompted a rigorous turn toward the view of techniques used in finance through methodical modeling of the volatility of financial assets. In addition, Francq and Zakoian (2010) stated that several augmentations of the GARCH model had been published, enabling new areas of research such as probability and statistics. Francq and Zakoian (2010) recommended the use of GARCH models as they are regarded as easy to use in empirical form and ample in theoretical problems. The recommendation is supported by Zakaria and Abdulla (2012), who also stated that GARCH models are the most efficient method to employ in modeling and predicting financial returns.

As many economies become progressively complex, the need to find a model that can accurately capture conditional variance and predict and estimate financial data arises. Bollerslev (1986) and Taylor (1986) recommended the use of GARCH models to capture volatility. The GARCH model developed by Bollerslev (1986) considered the lagged conditional variance terms as auxiliary regressors, which enabled more of a flexible lag structure. Additionally, ARCH/GARCH models regard heteroscedasticity as a variance to be modeled rather than an issue to be corrected (Engle, 2001).

Bollerslev (1986) developed the GARCH model to rectify the problem of over-parameterization often linked to the ARCH model. Bollerslev (1986) believed that the GARCH model would yield superior correlations compared to the ARCH model. Several studies have used the GARCH and GARCH-type models (such as the Exponential GARCH, Integrated GARCH, Threshold GARCH, etc.) to capture the asymmetric features of volatility. For instance, in South Africa, the symmetric and asymmetric GARCH models have been utilized to model financial and macroeconomic time series data (see Mpofo, 2016), Kutu and Ngalawa (2017), Babikir *et al.* (2012), etc.). Among the several techniques used to detect whether a cointegrating relationship exists among time series, the Johansen approach has been known to yield the most superior results (Kaltalıoğlu, 2010). Therefore, the paper employs the Johansen approach to determine whether cointegration among assets, capital, liabilities, and liquidity exists. Furthermore, the GARCH model is employed to model the volatility amongst the said variables.

The rest of the paper is organized as follows: section 2 presents the literature review, section 3 presents the research methodology, section 4 discusses the results of the study, and section 5 presents the conclusion.

## **LITERATURE REVIEW**

This section reviews the appropriate literature that employs the cointegration and the GARCH-type models in the analysis of time series data. Ziramba (2010) studied the price, and income elasticities of crude oil import demand in South Africa found using data covering the period 1980 to 2006. The study employed the Johansen cointegration technique. The results of the study

showed that there was a long-run cointegrating relationship among crude oil import demand, income, and price. An earlier study conducted by Ziramba (2008) on demand for residential electricity in South Africa used the bounds testing approach to cointegration with an AR dispersed structure for the period between 1978 and 2005. The results of that study revealed that electricity demand is determined by income in the long run.

Garza (2018) conducted a study to examine the relationship between poverty and economic growth in Mexico. The study employed data for the period 1960 to 2016. The study employed the Gregory-Hansen cointegration test, VECM, and the Granger causality test. The findings of the Gregory-Hansen cointegration test revealed that there is an existence of a short-run and long-run equilibrium relationship between poverty and economic growth. The VECM results suggested that, in the long run, a percentage increase in economic growth would result in a percentage increase in poverty reduction. Causality test results showed a bi-directional causality relationship between poverty and economic growth in Mexico.

Rahman and Barman (2018) conducted a study on a VECM approach to financial development, international trade, and economic growth in China after economic reform. The study used the Johansen cointegration and the VECM to assess the causal relationship among the variables. The Phillips-Perron (PP) unit root test was employed to test for stationarity and the order of integration. The results of the study showed that financial development, international trade, and economic growth were cointegrated, which implies the existence of a long-run relationship among the variables. The Johansen cointegration test revealed the existence of one cointegrating relationship among the studied variables. The results obtained from the VECM confirmed the long-run relationship between financial development, international trade, and economic growth in China. Rahman and Barman (2018) concluded that financial development is a driving force of economic growth in the short run and long run in China.

Investigations conducted by Menyah and Wolde-Rufael (2010), Gudan (2016), Mohammadi et al. (2018), as well as Seth and Sidhu (2018) showed the ability and efficiency of GARCH models in predicting and assessing volatility. Degiannakis and Floros (2010) recommended that employing the VECM-GARCH model significantly improves hedging.

Amusa *et al.* (2009) applied the bounds testing approach to cointegration within an ADLF for the assessment of aggregate demand for South African electricity for the period 1960 to 2007. The study by Amusa *et al.* (2009) found that social and economic reforms added to the quick increase in electricity usage in South Africa. Furthermore, the results revealed that income would be a key factor in electricity demand in the long run.

Mohammadi *et al.* (2018) assessed the effects of exchange rate volatility on foreign agricultural trade in Iran for the period 1980 to 2012. The real exchange rate volatility of the uniform structures, nonlinear, and the unsymmetrical GARCH were computed. The EGARCH coefficient was selected on account that it showed the existence of asymmetry in exchange rate volatility. To test for the cointegrating relationships between the variables, the Johansen Juselius VECM was applied. The estimates obtained from the model indicated that the exchange rate volatility had negative consequences on the exports of agricultural products in the long run. In addition to this, it was discovered that the GDP had a significant negative effect on agricultural products in the long run. Furthermore, the foreign income that was earned from exporting oil to other countries negatively affected the import and export of agricultural products. Mohammadi et

al. (2018) concluded that by reducing and selecting exchange rate volatilities within an acceptable boundary, favouring economic conditions could increase agricultural exports and also contribute to the future planning of production.

Ekong and Onye (2017) examined the financial volatility of daily stock returns in Nigeria over the period of 4<sup>th</sup> January 2012 to 13<sup>th</sup> August 2015. The study used the GARCH, exponential GARCH, threshold GARCH, and their augmented versions in the analysis of stock returns. The results revealed that the GARCH and the augmented EGARCH model computed using the generalized error distribution (GED) were the best-performing models.

The study by Choudry *et al.* (2018) employed several GARCH-type models: the bivariate GARCH, error correction model (ECM) GARCH, Baba, Engle, Kraft, and Kroner (BEKK) GARCH, dynamic conditional correlation (DCC) GARCH, GARCH with Cross-Sectional Volatility (GARCH-X), Glosten Jagannathan and Runkle (GJR) GARCH and GARCH with Jumps, to predict the Daily dynamic hedge ratios for Greece. The study utilized data spanning from 01/2000 to 07/2014. The results concluded that the GARCH model produced the most accurate estimates. Additionally, ECM-GARCH, GARCH-X, and GARCH-GJR established average confidence interval levels. It was also found that the GARCH-BEKK and GARCH-GJR models showed extreme forecast capability for the individual portfolio returns.

Babikir *et al.* (2012) conducted a comparison study that utilized the GARCH, MS-GARCH, and GJR-GARCH models to examine structural breaks and stock returns volatility. The study used time series data for the period 07/02/1995 to 08/25/2010. The results obtained from the study revealed that the GARCH model performed better than the MS-GARCH and GJR-GARCH models. The study concluded that structural breaks are essential features of volatility stock market returns and should, for this reason, be accounted for to improve forecasts of the volatility stock market returns.

By employing the GARCH (1,1) and EGARCH (1,1) models with the aid of financial time series data covering the period 1986 to 2013, Mpofu (2016) discovered that moving to a floating exchange rate system leads to increased volatility of the South African rand. In addition, the results indicated that the volatility of the other variables studied also contributed to the changes in the South African rand.

## **RESEARCH METHOD**

The paper employed secondary financial time series data obtained from the South African Reserve Bank (SARB). Monthly time series data for four variables, namely, assets, capital, liabilities, and liquidity, for the period 2005/02 to 2018/06 was utilized. Each variable consisted of 161 observations for the time period studied. The Eviews statistical software was utilized to obtain empirical results. Financial time series is non-stationary in nature; therefore, it is important to determine the order of integration of the variables.

The testing of a unit root in times series data has become a common procedure in macroeconomics and statistical analysis (Lopez, 1997). The paper employs the Augmented Dickey-Fuller (ADF) test, developed by Dickey and Fuller (1979), to test the times series properties of assets, capital, liabilities, and liquidity. Gervais and Khraief (2007) noted that the ADF test could be used to determine the existence of a unit root in time series data. The ADF test follows the null hypothesis stating that a unit root exists in the times series and an alternative hypothesis that the time series does not contain a unit root. The null hypothesis of the unit root can be rejected if the

p-value of the ADF test is less than the significance value. Hence, it can be concluded that the time series is nonstationary. The two hypotheses can be presented as follows:

$$H_0 : \theta = 1$$

$$H_1 : \theta < 1$$

The ADF unit root test was established using the following equation:

$$y_t = \beta' D_t + \phi y_{t-1} + \sum_{j=1}^p \psi_j \Delta y_{t-j} + \varepsilon_t \dots\dots\dots(1)$$

Where  $D_t$  is a vector of deterministic terms, the  $p$  lagged difference terms,  $\Delta y_{t-j}$  is used to estimate the autoregressive moving average structure of errors, the error term  $\varepsilon_t$  homoscedastic and is not correlated to  $p$ , the coefficients  $\beta'$ ,  $\phi$  and  $\psi_j$  are the estimates.

For determining the long-run relationship between the variables, the paper employs the Johansen (1991) cointegration methodology. Tsoku (2014) defined cointegration as a linear combination of two or more non-stationary time series that are stationary. Sohail and Hussain (2009) suggested the use of cointegration to examine long-run relationships among variables.

The Johansen methodology analyses three points: (i) estimating the number of cointegrating relationships in  $I(1)$  data, (ii) estimating the cointegrating relationships, and (iii) testing the economic hypotheses framework. The technique is based on the assumption that a VAR model defines the data sufficiently. The VAR model in the Johansen methodology is analyzed by employing likelihood methods to solve the three points mentioned above (Johansen, 1991). The Johansen method follows a VAR process of order  $p$  given by:

$$y_t = \mu + A_1 y_{t-1} + A_2 y_{t-2} + \dots + A_p y_{t-p} + \varepsilon_t \dots\dots\dots(2)$$

where  $y_t$  is an  $n \times 1$  vector of variables that are  $I(1)$  and  $\varepsilon_t$  is an  $n \times 1$  vector of innovations. The VAR equation shown above in equation (2) can be re-written as:

$$\Delta y_t = \phi + \Pi y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} + \varepsilon_t \dots\dots\dots(3)$$

where

$$\Pi = \sum_{i=1}^p A_i - I \text{ and } \Gamma_i = -\sum_{j=i+1}^p A_j$$

If the coefficient matrix  $\Pi$  has declined in rank  $r < n$ , then there exist matrices  $\alpha$  and  $\beta$  each with rank  $r$  such that  $\Pi = \alpha\beta'$  and  $\beta'y_t$  is  $I(0)$ . Where  $r$  represents the number of relationships that are cointegrated, the  $\alpha$  is the adjustment parameters in the vector error correction model, and  $\beta$  is a cointegrating vector in each column. For a given  $r$ , the maximum likelihood estimator of  $\beta$  describes the combination of  $y_{t-1}$  that yields the  $r$  largest canonical correlations of  $\Delta y_t$  with  $y_{t-1}$  when the lagged variations are corrected (Osterholm and Hjalmarsson, 2007).

The Johansen cointegration procedure provides the trace and maximum eigenvalue likelihood ratio test statistics. The maximum eigenvalue test statistic tests the adequacy of a single

cointegration equation. The rejection of the null hypothesis indicates the presence of at least one long-run relationship. The null and alternative hypotheses for the test are shown below:

$$H_0: r = r_0$$

$$H_1: r = r_0 + 1$$

The test statistic is as follows:

$$J_{max} = -T \ln(1 - \hat{\lambda}_{r+1}) \dots \dots \dots (4)$$

where  $T$  is the sample size,  $\hat{\lambda}$  is the  $i$ :th largest canonical correlation. The trace test examines the null hypotheses of a number of long-run relationships ( $r$ ) equal to a given value ( $r_0$ ) and the alternative hypothesis for  $r$  greater than  $r_0$ . The hypotheses are shown below:

$$H_0: r = r_0$$

$$H_1: r > r_0$$

The test statistic is given as:

$$J_{trace} = -T \sum_{i=r+1}^n \ln(1 - \hat{\lambda}_i) \dots \dots \dots (5)$$

Similar to the maximum eigenvalue test,  $r_0 = 0$ . The rejection of the null hypothesis shows that there is only one combination of the  $I(1)$  variables that will produce a stationary process.

The definition of GARCH methodology is constructed on the first two conditional moments, where the GARCH ( $p, q$ ) procedure ( $\epsilon_t$ ) is called a GARCH ( $p, q$ ) process if the first two conditional moments satisfy:

(i)  $E(\epsilon_t | \epsilon_u, u < t) = 0, t \in \mathbb{Z}$ .

(ii) There exist constants  $\omega, \alpha_j, i = 1, \dots, q$  and  $\beta_j, j = 1, \dots, p$  thus

$$\delta_t^2 = Var(\epsilon_t | \epsilon_u, u < t) = \omega + \sum_{i=1}^q \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2 \cdot (t \in \mathbb{Z}) \dots \dots \dots (6)$$

Compressing Equation (6) yields the equation below:

$$\sigma_t^2 = \omega + \alpha(B) \epsilon_t^2 + \beta(B) \sigma_t^2, (t \in \mathbb{Z}) \dots \dots \dots (7)$$

where  $B$  is the standard backshift operator  $B^i \epsilon_t^2 = \epsilon_{t-i}^2$  and  $B^i \sigma_t^2 = \sigma_{t-i}^2$  for any integer ( $i$ ), and  $\alpha$  and  $\beta$  are polynomials of degrees  $q$  and  $p$ , respectively:

$$\alpha(B) = \sum_{i=1}^q \alpha_i B^i, \quad \beta(B) = \sum_{j=1}^p \beta_j B^j$$

If  $\beta(z) = 0$

then

$$\sigma_t^2 = \omega + \sum_{i=1}^q \alpha_i \epsilon_{t-i}^2 \dots \dots \dots (8)$$

and the procedure is referred to as an ARCH( $q$ ) process. By definition, the innovation of the process  $\epsilon_t^2$  is the variable  $v_t = \epsilon_t^2 - \sigma_t^2$ . Substituting in equation (6) the variables  $\sigma_{t-j}^2$  by  $\epsilon_{t-j}^2 - v_{t-j}$ , the following representation is found as:

$$\epsilon_t^2 = \omega + \sum_{i=1}^q (\alpha_i + \beta_i) \epsilon_{t-i}^2 + v_t - \sum_{j=1}^p \beta_j v_{t-j}, (t \in \mathbb{Z}) \dots \dots \dots (9)$$

where  $r = \max(p, q)$ , with the convention  $\alpha_j = 0$  ( $\beta_j = 0$ ) if  $i > q$  ( $j > p$ ). This equation has a linear structure of an ARMA model, allowing for simple computation of the linear predictions. Under additional assumptions (implying the second-order stationarity of  $\epsilon_t^2$ ), it can be stated that if  $(\epsilon_t)$  is GARCH (p, q), then  $\epsilon_t^2$  is an ARMA (r, p) process. In particular, the square of an ARCH(q) process admits if it is stationary, an AR(q) representation will result. The ARMA representation is deemed to be useful for the estimation and identification of GARCH processes.

Many statistical techniques employed in the analysis of time series data make assumptions about normality, heteroscedasticity, serial correlation, t-tests, and analysis of variance (Mishra *et al.*, 2019). It is important to run diagnostic checks for these assumptions as it helps indicate whether the model applied in the study is adequate or not. Any efficient model should generate residuals that have a mean equal to zero, constant variance, and uncorrelated error. The heteroscedasticity of variance assumption is frequent in the application and should be inferred when computing any statistical test (Delacre *et al.*, 2017). Therefore, the underlying study employs the Lagrange multiplier (LM) to detect the presence or absence of heteroscedasticity. In addition, the Jarque-Bera (JB) test is utilised to determine whether the residuals applied in the study are normally distributed. Lastly, the Breusch-Godfrey (BG) test is employed to determine the presence or absence of serial correlation.

## FINDINGS AND DISCUSSION

The following section discusses the empirical results obtained from the data analysis. **Table 1** presents provide the descriptive statistics for the variables used in this paper.

Table 1. Descriptive statistics

	<b>L_AST</b>	<b>L_CAP</b>	<b>L_LIA</b>	<b>L_LIQ</b>
<b>Mean</b>	11.121	10.266	13.264	10.024
<b>Median</b>	11.017	10.283	13.327	9.893
<b>Maximum</b>	11.667	10.876	13.715	11.055
<b>Minimum</b>	10.070	9.243	12.628	8.957
<b>Std. Dev.</b>	0.423	0.341	0.286	0.618
<b>Skewness</b>	-0.528	-0.321	-0.528	0.360
<b>Kurtosis</b>	2.541	2.401	2.445	1.669
<b>JB test</b>	8.887	5.174	9.543	15.375
<b>Probability</b>	0.012	0.075	0.008	0.000
<b>Observations</b>	161	161	161	161

According to the results shown in **Table 1**, it is evident that liabilities have the highest mean value, which implies that the monthly changes in liabilities are of great significance compared to assets, capital, and liquidity. This shows that liabilities are very responsive to changes in contrast to the other variables. The standard deviations of assets, capital, liabilities, and liquidity are 0.423, 0.341, 0.286, and 0.618 respectively. This implies that liabilities have the highest degree of variation while capital has the lowest degree of variation.

The Jarque-Bera test reveals that the null hypothesis of normality is rejected for assets, liabilities, and liquidity, whereas capital data follows a normal distribution. The skewness statistic shows that the data assets, capital, and liabilities exhibit significant data points as compared to

those of liquidity. The kurtosis values for all the variables indicate that the four variables have flat peaks. **Figure 1** shows the graphical representation of assets, capital, liabilities, and liquidities at the level.

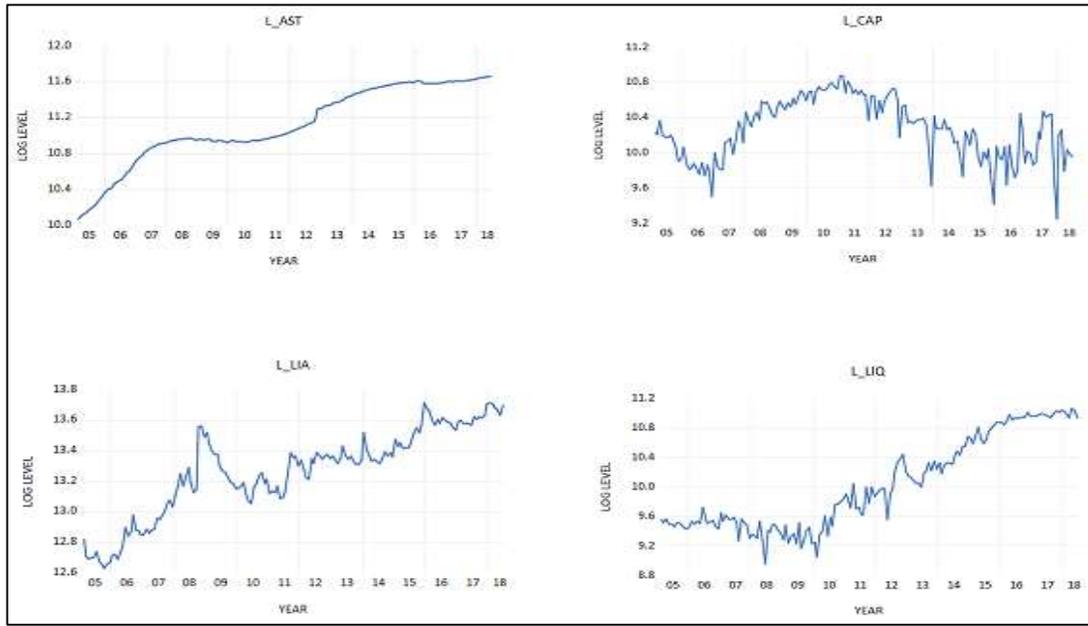


Figure 1. Log of Assets, Capital, Liabilities, and Liquidity at Level

According to **Figure 1**, it can be observed from the plot of assets that there has been a significant increase in an upward direction over the studied time sample; perhaps a cyclical pattern may be observed in the plot. The time series plot for capital shows a decline from late 2005 to early 2007. The trend then fluctuated in an upward direction, reaching a high of 10.9 in early 2011; thereafter, the trend declined, reaching a low of 9.2 in 2017. The liabilities plot depicts a long-run ascending plot with extreme fluctuations for the observed time period. It is observed from the liquidity plot that the series exhibits an irregular upward pattern with sharp declines in 2008, 2009, and early 2012. By eye inspection, the four-time series plots appear to be non-stationary. **Figure 2** presents the plot of the variables at first difference

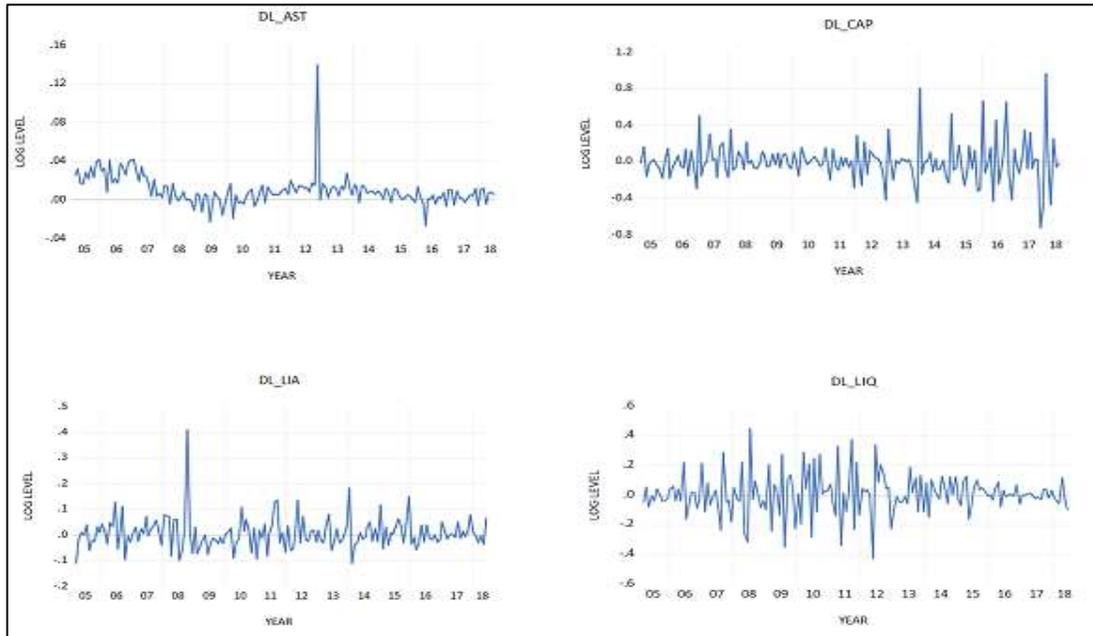


Figure 2. Log of Assets, Capital, Liabilities, And Liquidity at First Difference

**Figure 2** shows the graphical representation of assets, capital, liabilities, and liquidity at first difference. The differenced time series plots for the four variables depict a constant mean and variance. By eye inspection, one may conclude that the time series plots appear stationary. The results of the formal test of stationarity are presented in **Table 2**.

Table 2. Unit Root Tests for L\_AST, L\_CAP, L\_LIA and L\_LIQ

Variables	Order of integration			
	I(0)		I(1)	
	Test statistic	P-value	Test statistic	P-value
L_AST	-2.546	0.107	-3.896	0.003
L_CAP	-1.963	0.303	-15.629	0.000
L_LIA	-1.368	0.597	-13.231	0.000
L_LIQ	-0.400	0.905	-19.751	0.000

**Table 2** shows a summary of the results obtained from the ADF test. According to the results, it is revealed that all variables are non-stationary at the level and stationary after the first difference. Hence it can be concluded that all the variables are integrated into order 1  $I(1)$ .

In view of the fact that the variables are integrated of order one or  $I(1)$ , cointegration analysis can then be employed. The test to determine the optimal lag order in the VAR was conducted and the results presented in **Appendix I** suggested that lag 3 was the most optimal lag. The trace and maximum eigenvalue maximum likelihood tests for Johansen cointegration methodology were conducted based on the VAR order of 3. **Table 3** presents the trace test, and the results indicate the presence of one cointegrating relationship among the variables. **Table 4** presents the results

obtained from the maximum eigenvalue test, and the results show that no cointegrating relationship exists among the variables.

Table 3. Trace Test

Hypothesized no. of CE(s)	Eigenvalue	Trace Statistic	Critical value	Prob.**
<b>None*</b>	0.137	50.034	47.856	0.031
<b>At most 1</b>	0.086	26.826	29.797	0.106
<b>At most 2</b>	0.076	12.642	15.495	0.129
<b>At most 3</b>	0.001	0.222	3.841	0.637

Note: Trace test indicates 1 cointegrating eng(s) at the 0.05 level, \*denotes rejection of the hypothesis at the 0.05 level, \*\*Mackinnon-Haug-Michelis (1999) p-values.

Table 4. Maximum Eigenvalue

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	Critical value	Prob.**
<b>None</b>	0.137	23.207	27.584	0.165
<b>At most 1</b>	0.086	14.184	21.132	0.350
<b>At most 2</b>	0.076	12.420	14.265	0.096
<b>At most 3</b>	0.001	0.222	3.841	0.637

Note: Max-eigenvalue test indicates no cointegration at the 0.05 level, \*denotes rejection of the hypothesis at the 0.05 level, \*\*MacKinnon-Haug-Michelis (1999) p-values.

It could be hard to explain contradicting results obtained from the Trace test and Maximum eigenvalue test; nonetheless, elective actions could be employed to assess the degree of cointegration that exists among the variables (Maggiore and Skerman, 2009). Further tests in the underlying study are carried out on the presumption that the results yielded by the Trace test are correct. This is based on the assumption made by Lütkepohl *et al.* (2001), which stated that situations occur where results yielded from the Trace test are considered superior to the Maximum eigenvalue tests. **Table 5** presents the results of the cointegrating vector of the series.

Table 5. Cointegrating Vector for L\_AST, L\_CAP, L\_LIA and L\_LIQ

<b>1 Cointegrating Equation(s)</b>	<b>Log-likelihood</b> 856.521		
<b>Normalized cointegrating coefficients (standard error in parenthesis)</b>			
L_AST	L_CAP	L_LIA	L_LIQ
1.000	0.291 (1.795)	-15.560 (3.419)	3.386 (1.478)

The long-term equilibrium vector is estimated to be  $Z = L\_AST + 0.291 L\_CAP - 15.560 L\_LIA + 3.386 L\_LIQ$ . The coefficient of L\_CAP has a standard error of 1.759 and is not significant. The coefficient of L\_LIQ has a standard error of 1.478 and is significant. The coefficient of L\_LIA has a standard error of 3.419 and is also significant. L\_AST is denoted as the dependent variable, and L\_CAP, L\_LIA, and LIQ are denoted as the independent variables. Therefore, in the long run, L\_LIA has a positive impact on L\_AST while L\_CAP and L\_LIQ have a negative impact on L\_AST, on average, *ceteris paribus*. Additionally, for every 1% increase in assets acquired, liabilities rise by 15.56 % in the long run. In conclusion, the null hypothesis of no cointegrating relationship in the model is

rejected. The results are in line with the study by Amusa *et al.* (2009). The paper is also supported by the study by Menyah and Wolde-Rufael (2010).

The results presented in **Appendix II** show the short-run and adjustment coefficients. According to the results, the error correction equation signifying the long-run relationship among the variables is shown below:

$$ECT_{t-1} = 1.000L\_AST_{t-1} + 0.292L\_CAP_{t-1} - 15.560L\_LIA_{t-1} + 3.386L\_LIQ_{t-1} + 158.420.....(10)$$

According to the results obtained, the previous year's deviation from long-run equilibrium is corrected in the current period as an adjustment speed of 0.02, -0.00, 0.007, and -0.00 for assets, capital, liabilities, and liquidity respectively. A percentage change in L\_CAP is associated with a 0.002% decrease in L\_AST on average, *ceteris paribus*, in the short run. A percentage change in L\_LIA is associated with a 0.017% increase L\_AST on average, *ceteris paribus* in the short run. A percentage change in L\_LIQ is associated with a 0.02% increase in L\_AST on average, *ceteris paribus*, in the short run. An examination of the short-run relationship in the assets regression showed that assets are affected by the second lag of assets ( $\alpha_{2,1} = 0.17, z = 2.061$ ) and the third lag of liquidity ( $\alpha_{5,3} = 0.02, z = 2.172$ ), which are significantly positive, and the second lag of capital ( $\alpha_{3,2} = -0.014, z = -2.356$ ) which is significantly negative. The rest of the coefficients estimated in this equation are not significantly different from zero.

In capital regression, the effects of the first lag of assets ( $\beta_{2,1} = -2.763, z = -2.387$ ), the first and second lag of capital ( $\beta_{2,1} = -0.448, z = -4.990, \beta_{2,2} = -0.097, z = -5.424$ ) are significantly positive. For the Liabilities equation, the short-run effects of capital in lag two ( $\phi_{2,2} = -0.057, z = -2.222$ ) are significantly negative; the other coefficients are not significantly different from zero. Lastly, for the liquidity equation, the short-run effects of liquidity in the first lag ( $\gamma_{2,1} = -0.456, z = -5.313$ ) are statistically insignificant and negative. The diagnostic tests for the estimated vector error correction model are discussed in **Table 6, Table 7, and Table 8.**

Table 6. VECM Joint Residual Heteroscedasticity Tests

Chi-sq	df	Prob.
286.9128	260	0.1208

The results for the heteroscedasticity test shown in **Table 6** are used to test for conditional heteroscedasticity. The chi-square probability value is 0.121, which is greater than the 0.05 critical value; therefore, the null hypothesis cannot be rejected. It can be concluded that the model is not heteroscedastic. The LM test is performed to detect the presence or absence of serial correlation in the estimated residuals. The serial correlation LM test for the estimated VECM is shown in **Table 7.**

Table 7. VECM Residual Serial Correlation LM Tests

Null hypothesis: No serial correlation at lag h						
Lag	LRE* stat	Df	Prob.	Rao F-stat	df	Prob.
1	16.19911	16	0.4392	1.015000	(16, 416.1)	0.4393
2	22.21812	16	0.1363	1.402151	(16, 416.1)	0.1364
3	18.87760	16	0.2751	1.186604	(16, 416.1)	0.2752

**Null hypothesis: No serial correlation at lags 1 to h**

Lag	LRE* stat	Df	Prob.	Rao F-stat	df	Prob.
1	16.19911	16	0.4392	1.015000	(16, 416.1)	0.4393
2	29.40982	32	0.5983	0.918233	(32, 488.4)	0.5988
3	47.96090	48	0.4744	1.000419	(48, 495.1)	0.4758

\*Edgeworth expansion corrected likelihood ratio statistic.

The Breusch-Godfrey, serial correlation LM test is used to test the presence and/or absence of serial correlation in the residuals. The null hypothesis can be rejected since the critical p-value of 0.05 is less than the probability values estimated in **Table 7**. Thus, it can be concluded that there is no serial correlation in this model. The Jarque-Bera test is utilised in the study to assess whether the variables within a model follow a normal distribution or deviate from a normal distribution. The results of the Jarque-Bera test are shown in **Table 8**. A critical value of 0.05 is compared to the probability value of the Jarque-Bera statistic for the formation of a conclusion.

Table 8. VECM residual normality test

Component	Jarque-Bera	Df	Prob.
1	19749.11	2	0.0000
2	95.11431	2	0.0000
3	868.3048	2	0.0000
4	7.352000	2	0.0253
<b>Joint</b>	20719.89	8	0.0000

\*Approximate p-values do not account for coefficient estimation

As seen in **Table 8**, components 1-4 have a probability value that is less than the 0.05 critical value; thus, assets, capital, liabilities, and liquidity do not follow a normal distribution. The joint Jarque-Bera statistic is 20719.89, and the probability value is 0.000. Since the probability values are less than 0.05 level of significance, the null hypothesis is rejected. It is therefore concluded that the overall model does not follow a normal distribution. The paper is in line with Mohammadi *et al.* (2018) as well as Seth and Sidhu (2018). With the long-run relationship established, the next step is to model the volatility.

Prior to estimating the ARCH and GARCH model, the ARCH LM test was employed to test for the presence of ARCH effects in the residuals. The results are summarised in **Table 9**.

Table 9. Heteroscedasticity Test (ARCH)

F-statistic	62.418	Prob. F (3,154)	0.000
Obs*R-squared	86.698	Prob. Chi-Square (3)	0.000

The results obtained rejected the null hypothesis of no ARCH effect at 5% significance level. Therefore, ARCH effects are present in the model. Hence, the ARCH/ GARCH model can be estimated. **Table 10** presents the results for the ARCH/ GARCH model.

Table 10. GARCH (1,1) estimation

<b>GARCH = C(5) + C(6)*RESID(-1)^2 + C(7)*GARCH(-1)</b>				
<b>Variable</b>	<b>Coefficient</b>	<b>Std. Error</b>	<b>z-Statistic</b>	<b>Prob.</b>
C	3.370	0.413	8.168	0.000
L_CAP	0.009	0.007	1.253	0.210
L_LIA	0.323	0.035	9.355	0.000
L_LIQ	0.343	0.009	36.481	0.000
<b>Variance Equation</b>				
C	0.000	0.000	0.826451	0.408
ARCH	0.575	0.177	3.245	0.001
GARCH	0.515	0.087	5.944	0.000
R-squared	0.755	Mean dependent var		11.121
Adjusted R-squared	0.750	S.D. dependent var		0.424

Durnel (2012) recommends the use of the Student’s t-statistic over the Gaussian normal distribution as it yields superior in-sample results. The GARCH model was then estimated using the Student’s t-statistic. According to the results observed in **Table 10**, the GARCH equation is estimated as:

$$L_{AST} = 3.370 + 0.009L_{CAP} + 0.323L_{LIA} + 0.343L_{LIQ} \dots\dots\dots(11)$$

**Equation (11)** illustrates the GARCH (1,1) model. The results show that capital, liabilities, and liquidity have a positive relationship with assets, suggesting that a percentage increase in capital, liabilities, and liquidity would result in a percentage increase in assets. According to the results in **Table 10**, the constant value, which explains the long-run variation in the model, has a significant p-value of 0.408. The ARCH statistic has a significant p-value of 0.001, which is less than the 5% level of significance. Similarly, the GARCH effect has a significant value of 0.000. The results suggest that the previous year's data and internal changes have an impact on changes in assets. Additionally, the GARCH coefficient indicates that disturbances to volatility have a continued effect on conditional variance. The coefficients of ARCH and GARCH are >1, implying that shocks decay rapidly in the future. It also suggests a great presence of ARCH and GARCH effects (Bonga, 2019). The diagnostic tests of the GARCH model are presented in **Table 11, Table 12, and Figure 3**.

Table 11. Heteroscedasticity Test (ARCH)

<b>Heteroscedasticity Test: ARCH</b>			
<b>F-statistic</b>	0.777	<b>Prob. F(3,154)</b>	0.508
<b>Obs*R-squared</b>	2.356	<b>Prob. Chi-Square(3)</b>	0.502

Table 12. Serial correlation

<b>Autocorrelation</b>	<b>Partial Correlation</b>	<b>AC</b>	<b>PAC</b>	<b>Q-Stat</b>	<b>Prob*</b>	
. .	. .	1	0.005	0.005	0.0044	0.947
. .	. .	2	-0.025	-0.025	0.1096	0.947
* .	* .	3	-0.119	-0.119	2.4612	0.482
<b>*Probabilities may not be valid for this equation specification</b>						

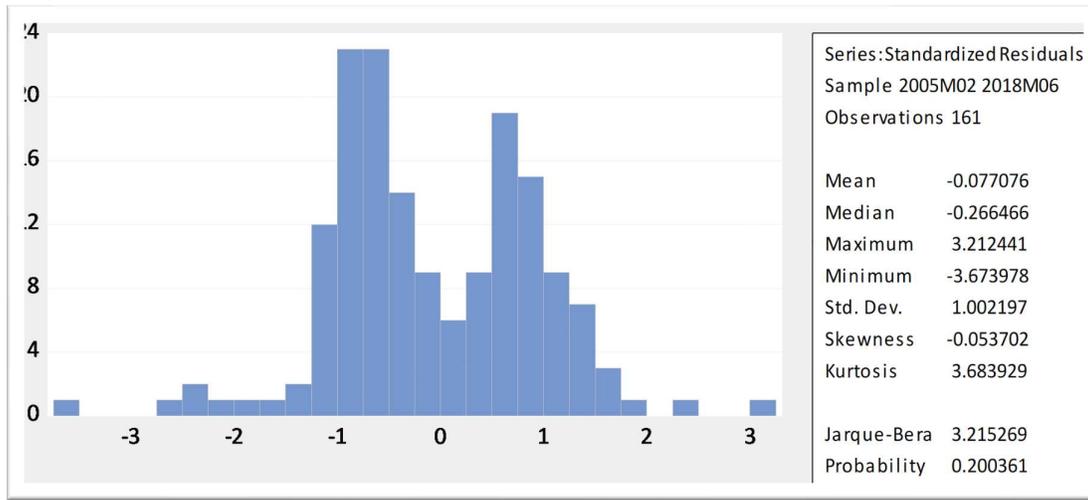


Figure 3. Test for normality in GARCH model

The diagnostic tests reported in **Table 11**, **Table 12**, and **Figure 3** revealed that equation (11) possesses the characteristics of a good model. The heteroscedasticity test presented in **Table 11** cannot reject the null hypothesis of no ARCH effects in the residuals. Therefore, the model does not have ARCH effects. Similarly, the serial correlation test shown in **Table 12** indicates that the model is not affected by serial correlation. **Figure 3** presents the Jarque-Bera test with a probability value greater than 5% level of significance; hence it can be concluded that the data follows a normal distribution.

## CONCLUSION

The underlying paper employed the Johansen cointegration methodology and the GARCH model to analyze the relationship and the volatility between assets, capital, liabilities, and liquidity for the period spanning from 2008/06 to 2018/02. The results obtained from the ADF test suggest that assets, capital, liabilities, and liquidity are integrated of order 1. The results from the Johansen cointegration test revealed that at least one cointegrating relationship exists among the four financial variables. The findings are consistent with the results obtained by Rahman and Barman (2018). Since it was found that the time series were cointegrated, this means that assets, capital, liabilities, and liquidity are related and can be combined linearly. Additionally, short-run shocks that may affect movements within the individual time series would converge in the long run. Furthermore, results suggested that in the long run, liabilities have a positive impact on assets while capital and liquidity have a negative impact on assets, on average, *ceteris paribus*. It was also found that for every 1% increase in assets acquired, liabilities rise by 15.56% in the long run. Thus, the null hypothesis of no cointegrating relationship in the model was rejected. The results obtained from the VECM indicated that the previous year's deviation from long-run equilibrium is corrected in the current period as an adjustment speed of 0.02, -0.00, 0.007, and -0.00 for assets, capital,

liabilities, and liquidity respectively. The diagnostic tests revealed that the estimated cointegration model is adequate.

The results from the ARCH test show that the ARCH effect has a statistically significant probability value. Similarly, the computed GARCH model has a significant p-value. This implies that the previous year's statistics and internal changes contribute to the volatility of assets. The results suggest that capital, liabilities, and liquidity have a positive relationship with assets. It was also observed that the coefficient of the ARCH and GARCH adds up to a value greater than one, which indicates that shocks decay more quicker in the future. Since the sum of the coefficients is a value greater than one, it implies that the conditional variance is explosive. The findings are similar to those found by Bonga (2019), where the estimated GARCH model generated statistically significant coefficients at 5% significant level with an explosive conditional variance. The diagnostic tests revealed that the estimated ARCH/GARCH model displays characteristics of a well-specified model.

#### **LIMITATION & FURTHER RESEARCH**

Although the estimated models were well-specified, there is room for further improvement. This paper recommends the use of other GARCH-type models for analysis. Additionally, the paper only makes use of the Student's t-distribution as recommended by Durnel (2012); therefore a study exploring the comparison performance of the other error distributions could yield differing results from those obtained in the underlying study. Furthermore, an alternative cointegration methodology could be utilized, and more financial or macroeconomic variables could be employed in a similar analysis.

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**APPENDIX**

## Appendix I. VAR Lag Order Selection Criteria

Lag	LogL	LR	FPE	AIC	SIC	HQ
0	1.969	NA	0.000	0.014	0.073	0.038
1	679.198	1319.048	3.27e-08	-8.722	-8.484*	-8.625
2	691.582	23.635	3.13e-08	-8.766	-8.350	-8.597
3	710.374	35.128*	2.76e-08*	-8.894*	-8.300	-8.652*
4	715.260	8.943	2.91e-08	-8.840	-8.068	-8.526
5	720.200	8.846	3.07e-08	-8.787	-7.836	-8.401
6	723.963	6.592	3.30e-08	-8.718	-7.589	-8.260
7	731.093	12.209	3.38e-08	-8.694	-7.387	-8.163
8	735.738	7.772	3.59e-08	-8.637	-7.152	-8.034

\* indicates lag order selected by the criterion, LR: sequential modified LR test statistic (each test at 5% level)

FPE: Final prediction error, AIC: Akaike information criterion, SC: Schwarz information criterion, HQ: Hannan-Quinn information criterion.

## Appendix II. Vector Error Correction Estimates

<b>Cointegrating Eq:</b>	<b>CointEq1</b>			
L_AST(-1)	1.000			
L_CAP(-1)	0.291[ 0.162]			
L_LIA(-1)	-15.560[-4.551]			
L_LIQ(-1)	3.386[ 2.307]			
C	158.420			
<b>Error Correction:</b>	D(L_AST)	D(L_CAP)	D(L_LIA)	D(L_LIQ)
<b>CointEq1</b>	0.002[ 3.701]	-0.000[-0.018]	0.007[ 2.808]	-0.000[-0.053]
D(L_AST(-1))	0.057[ 0.7]	-2.763[-2.387]	-0.485[-1.339]	-1.023[-1.354]
D(L_AST(-2))	0.170[ 2.061]	-0.210[-0.181]	-0.131[-0.362]	-0.513[-0.680]
D(L_AST(-3))	0.086[ 1.035]	1.129[ 0.965]	-0.451[-1.233]	0.614[ 0.805]
D(L_CAP(-1))	0.000[ 0.071]	-0.418[-4.990]	-0.047[-1.799]	-0.044[-0.804]
D(L_CAP(-2))	-0.014[-2.356]	-0.448[-5.424]	-0.057[-2.222]	0.033[ 0.606]
D(L_CAP(-3))	-0.002[-0.333]	-0.097[-1.128]	-0.028[-1.023]	0.063[ 1.128]
D(L_LIA(-1))	0.009[ 0.498]	0.058[ 0.220]	-0.033[-0.399]	0.045[ 0.259]
D(L_LIA(-2))	-0.003[-0.163]	0.239[ 0.922]	-0.060[-0.739]	0.269[ 1.587]
D(L_LIA(-3))	0.017[ 0.932]	-0.078[-0.303]	-0.073[-0.901]	-0.073[-0.431]
D(L_LIQ(-1))	-0.002[-0.235]	-0.173[-1.320]	-0.030[-0.740]	-0.456[-5.313]
D(L_LIQ(-2))	0.011[ 1.087]	-0.120[-0.854]	0.021[ 0.485]	-0.140[-1.525]
D(L_LIQ(-3))	0.020[ 2.172]	-0.198[-1.572]	0.050[ 1.257]	-0.137[-1.666]
C	0.006[ 3.740]	0.018[ 0.757]	0.017[ 2.382]	0.024[ 1.553]

**Econometric Modelling of Financial Time Series**

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R-squared	0.358	0.313	0.137	0.253
Adj. R-squared	0.299	0.250	0.059	0.185
S.E. equation	0.013	0.188	0.059	0.123

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