

Examining Challenges Associated with Numerical Cognition in Early Years Challenges Associated with Numerical Cognition in Early Years

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Abstract

The current research aims at examining challenges associated with numerical cognition in the early years of South Africa. Guided by cognitive learning theory, the current investigation was informed by a continuous quest in South Africa. The aim was to respond to unresolved challenges associated with early numerical cognition in terms of numerical cognition through semantics and textual misunderstanding in early numerical problems and concepts. Using survey design, a sample of 80 learners was chosen and tested through descriptive statistics. Data was collected using semi-structured questionnaires. It was revealed that the challenges associated with numerical cognition in early numerical problems and concepts are counting, reading numbers that contain more than one digit, difficulties with copying numbers, mathematical signs confusion, and challenges associated with manipulatives. By implication, the study highlighted that there is a severe lack of numerical literacy and competency among learners. Implying too those teachers need to pay particular attention to both semantics and textual misunderstanding.

Keywords: *Numerical cognition, foundation phase, number concepts, South Africa*



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INTRODUCTION

Over the past decades, research suggested that numerical cognition, with reference to semantics and textual misunderstanding in early childhood, numerical concepts still remain a concern (Botha 2011; Chernyak et al., 2019; Chernyak et al., 2016; Cockburn, 1999; Cotton, 2010; Kadja 2010; Pournara et al., 2016). Even though a successive number of studies conducted prior to Chernyak et al.'s (2019) work have attempted to find solutions to the challenges associated with numerical cognition in the foundation phase, there has not been any clear evidence of solution(s) (Botha, 2011; Blake et al., 2015; Cotton, 2010; Kadja, 2010). Studies that laid an emphasis on the understanding of the mathematical text also could not provide any demonstrable evidence to resolve the identified challenges (Askew, 2018; Blake et al., 2015; Chernyak et al., 2019; Gray & Reeve, 2016; McCrink & Spelke, 2016; Maloney 2011; Murray, 2011; Pardesi, 2008; Pournara et al., 2016; Xiao et al., 2019). Despite such a lack of solutions, the challenges remain, and Pournara et al. (2016, p. 1) highlight that across many countries, there is "...much interest in learners' [cognition of] mathematics for the past 30 years...." Earlier studies had focused more on constructivist perspectives (Ben-Zeev, 1998; Borasi, 1994; Olivier, 1989; Radatz, 1979). Others examined the socio-cultural and discursive stance (e.g., Brodie & Berger, 2010; Ryan & Williams, 2007). Consequently, the challenges experienced in both foundation and senior phases still linger. This is particularly true in South Africa in pursuit of solutions, particularly over the past five to ten years, and consequently, the "... resurgence in focus on learner..." cognition of mathematical concepts (Pournara et al., 2016, p. 2). In response to such persistently occurring challenges regarding numerical cognition, the current study sought to explore early numerical problems and concepts. The investigation partly involved a review of literature based on the relationships underlying mathematical concepts and the research underscoring the findings or identifying gaps thereof. Much of the forthcoming analysis was based on the examination of the literature on numerical conceptualisation, with distinctiveness to early number concepts, consequently being underscored by the cognitive leaning learning theory.

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Theoretical framework – Cognitive learning theory

The need to know concepts and how one knows the concept has some time now been used in examining mathematics cognition (Brodie, 2014; Brodie & Berger, 2010; Herholdt & Sapire, 2014; Makonye & Luneta, 2014; Shalem et al., 2014). For instance, Herholdt and Sapire (2014) explain that knowing how a concept interlinks with other concepts is what cognitive learning theory entails. Thus, through cognitive learning theory, the assertion that the semantics of mathematical text play a role in early numerical cognition may not be disputed (Chernyak et al., 2019; McCrink & Spelke, 2016; Moore et al., 2016). These studies included, for instance, research conducted on student learning and collaborative mathematics (Webb, 2003); the assessment of quality education in early childhood (Weber, 1990); conceptual sources of the verbal counting principles (Carrey, 2007); pupils' cognition of numbers (Gelman & Gallistel, 2005); influences of the structure and learning process of mathematics (Niss, 2006); knowledge and understanding of numbers (Jarvin, 2009); and a study on learning to understand arithmetic (Kadja, 2010). Conclusions reached from examining the aforementioned studies are essential that the challenges include (1) calculation problems, (2) counting objects, and (3) number line comprehension. Regarding the calculation problems, for instance, the assertion via cognitive learning theory is based on the need for learners to achieve cognition of the numerical magnitude and its representation.

Despite such conclusion, the processing of numerical magnitude in learners, which influences the improvement of counting, still lacks conclusive evidence in research (Chernyak et al., 2019; McCrink & Spelke, 2016; Sousa, 2008; van Marle & Geary, 2016). Even though efforts had been made to resolve the inconclusiveness regarding the processing of numerical magnitude in learners, there had only been a suggestive stance taken by Radatz (2005) on this topic. Consequently, for many learners, the learning of mathematical concepts, symbols, and vocabulary remains a 'foreign language' problem (Bornman, 2010; Chernyak et al., 2019; McCrink & Spelke, 2016; Moore et al., 2016). As a direct consequence of the findings highlighted in these studies, errors are seen to be a function of other unknown variables in a misunderstanding of the mathematical text and process (Bornman, 2010; Wörle & Paulus, 2018; Ziv & Sommerville, 2017). On the other hand, too, various positions have been held by cognitive learning theory scholarship regarding what should be recognised as [mis]understanding of the semantics of the mathematical text (Bornman, 2010; Burrows, 2000; Buswell, 1999; Flavell, 1999; Goswami, 2008; Kim, S. & Kim, 2016; Piping, 2001; Shalev, 2004; Shunkoff, 2000; Wörle & Paulus, 2018; Ziv & Sommerville, 2017). For instance, based on Goswami's (2008) view, challenges associated with understanding of mathematical text may be the result of deficits in basic prerequisites, including unfamiliarity with algorithms/procedures and an unsatisfactory fundamental knowledge of mathematical concepts. In response to Buswell (1999), Piping (2001) and later Ziv and Sommerville (2017) offered a categorisation of what should also be used as a gauge in evaluating misunderstanding of the semantics in mathematical text and consequently its cognition. For instance, there could be an error of association – which may involve unfamiliar algorithms/procedures. There could also be the error of interference in which different operations or concepts interfere with each other; for example:

$$\left\{ \begin{array}{r} 845 \\ +372 \\ +1217 \\ +561 \\ - - - \\ 1778 \end{array} \right.$$

In an attempt to explain various errors associated with the calculation above, the concept of *place value* has not fully been understood. As such, the learner tends to combine two place values, "7" and "1" for instance, while neglecting the need for and magnitude of other values. Such misunderstanding or interference of concepts may lead to various errors, as exemplified. Notwithstanding the

assessment offered by Flavell (1999), Shalev (2004), Shunkoff (2000), and Skott (2005) opine that when one attempts to go beyond the description of faulty techniques and error patterns and towards the analysis of possible causes in the learners' cognitions, the various aspects of information processing seem to offer a good basis for classification. This has also been suggested by Kim, S., and Kim (2016), Wörle and Paulus (2018), and Ziv and Sommerville (2017). Most significantly, though, was the search for answers regarding the conceptual sources of verbal counting principles as well as how early childhood mathematics learning knowledge construction is formed, first by Edwards (2000), later by Carrey (2007), and thereafter by a number of authors (Ekdahl et al., 2016; Liu et al., 2015; Shou et al., 2015) as further examined in related work.

Related work

Between the years 2000 and 2013, several studies were conducted on this topic (Abadzi, 2006; Siegler & Thompson, 2005; Wright, 2000 Gray & Reeve, 2016; Xiao, et al., 2019; Yang et al., 2011). For instance, Gray and Reeve (2016) raised the need for number-specific and general cognitive markers. This is because the authors established an improved performance in pre-schoolers' math ability profiles (Gray & Reeve, 2016). Additionally, Xiao et al. (2019) established that cognition impairment prior to errors could be improved through working memory, which is based on event-related potentials. Similarly, Yang et al. (2011) concluded that error pattern analysis of elementary school-aged students improves limited English proficiency.

The first general conclusion these authors drew was that educators would be able to support learners by using the vocabulary related to addition and subtraction and working towards recording their addition and subtraction calculations using number sentences. Second, educators may introduce the symbols to record their practical activities rather than ask them to calculate from number sentences such as $7 + 5 = ?$ What is meant through the work of Xiao et al. (2019), as indicated earlier, is that working memory could be improved via event-related potentials. For instance, educators may develop grouping activities in the foundation stage by asking learners to combine groups of 2, 5, or 10 objects and by sharing them into equal groups. From the findings of these studies, a conclusion was drawn that by the end of school year 1, learners may be able to add or subtract one-digit numbers or multiples of 10 to one-digit or two-digit numbers. They may also use informal written methods to support them in these calculations. During school year 2, educators will be expected to introduce learners to four operations (symbols), +, -, x, and / as well as =. By the end of the year, learners would be expected to be able to find unknown numbers in number sentences such as $20 - ? = 12$. In conclusion, it can then be expected that learners would be able to add and subtract one-digit numbers or multiples of 10 from two-digit numbers, and they may use practical or informal methods to add and subtract two-digit numbers. If this has been achieved, then learners can understand that addition and subtraction are inverse operations and will be able to represent multiplication as an array; for example, 4×3 is the same as Figure 1. For instance, considering the lines by columns, this scheme represents 4×3 ; considering the columns by the lines, this schema represents 3×4 . Consequently, it might be used to show the commutability of the multiplication.

			
			
			
			

Figure 1: Multiplication table using objects

Another conclusion that can be drawn from such prior education approaches is that the selected approach can be a practical and informal method to support multiplication and division, including finding remainders. By the end of year 3, for instance, learners will be expected to add or subtract one-digit and two-digit numbers mentally. This also means that learners will have developed the ability to use a range of written methods to record and explain the addition and subtraction of two-digit and three-digit numbers. An important consequence is that educators need to introduce learners to practical and informal approaches to multiplying and dividing two-digit numbers and teach them that multiplication and division are inverse operations, laying the basis for learners in school year 3, starting to find unit fractions. *Case in point 1*: The introduction by using partitioning, for example: $39 + 52 = 30 + 50 + 9 + 2 = 80 + 11 = 91$.

In conclusion, it is essential to recognise that the perceptual method divides between educators who cling to the comfort of counting procedures and those who find more effective methods. Counting procedures, at best, enable learners to solve simple problems by counting. It also separates those who develop a more flexible form of arithmetic, where at best, the symbols can be used dually as processes or as concepts to be manipulated mentally. Thus, perceptual thinking occurs when counting procedures are compressed into number concepts with a rich connection. For example, knowing that "adding 4 and 2 makes 6, so 6 take away 4 must be 2"; and then by using this knowledge to derive new knowledge where "26 take away four is 22 because 26 is just 20 and 6", means that until learners can make the shift from process to the concept, they will not be able to understand that 10 is a concept, and they will not be able to comprehend two-digit numbers and place value. These challenges form the basis of the current research questions. What could be drawn from the cases is that most of the strategies of numerical error analysis in the foundation phase will remain hidden unless teachers make a specific effort to uncover these strategies. This also implies that numerical error analysis strategies are opportunities for the delivery of effective teaching because numerical error analysis is a function of errors or misconceptions. Goswami (2008) suggested three types of mathematical concept that correlates with numerical error analysis; (1) based on the perception of objects, (2) based on the processes that are symbolised and conceived dually as process or object, and (3) based on a list of properties that act as a concept. Wright (2000) attempted to offer a relative opinion and suggested that during enumeration, while the majority of 10-year-olds could solve problems, some experienced difficulty in the structural understanding of combinatorial problems. The conclusion drawn was that these "grabbers" did not yet understand the counting sequence and regarded it as a meaningless verbal sequence. Some authors suggested solutions such as found in the works of Alex (2002) and Edward (2000), or Scholar (2008) and Taylor (2006), which led to a conclusion that suggests significantly low numeracy competencies and lack of an adequate foundation for some children in understanding a mathematical text. Amidst the gaps identified in previous studies and partially unclear and not yet established findings, the current research sought to pursue the following research question:

Research questions

Based on the introduction and related work, the following research question guided the formulation of the rest of the research: What numerical concepts in early childhood numerical cognition remain a concern in the foundation phase?

Methodology

General Background

In seeking to explore the challenges associated with mathematical concepts in the early years, the researchers collected data by using semi-structured questionnaires with questions related to numerical errors in the foundation phase. A critical procedure considered was that learners had to be assisted by one teacher per school when trying to answer the statements in all 16 schools. Recognising the various knowledge gaps and challenges highlighted in the introduction as well as related work, the method for the study was guided by the question of what mathematical concepts in early

numerical problems and concepts are a challenge and influence numerical cognition in the foundation phase.

Participants and instrumentation

The participants in this study were foundation phase in South Africa. Sixteen schools were observed. Attention was focused on mathematical concepts in early numerical problems and concepts that are a challenge and influence numerical cognition in the foundation phase. For instance, the learners worked through counting concepts. All sixteen schools' participants were of the same teaching and learning infrastructure, instructed by the same teacher, and had experienced similar learning activities. Throughout the investigation, all learners were also from the same schools system and had all taken the same foundation phase programme in previous years. The concepts attempted by each school and thus class is identifiable and assisted by the class teachers and the research team. These concepts generally included but were not limited to; using these overarching themes. The following subthemes were formed; I always make mistakes when counting, I have a problem in reading numbers that contain more than one digit, I have problems when copying the numbers, mathematical signs confuse me, I always finish my tasks, I learn the best when using different kinds of colours, I have problems understanding mathematical language. The method used in this research was a 4-point Likert scale, which required that learners either strongly agreed, agreed, disagreed, or strongly disagreed with a statement in the questionnaire. Thus, statements were used to investigate the relationships underlying numerical errors. In each school, a teacher was available to help learners complete the questionnaire as indicated in the general background of methodology.

Sample selection

The sample size of 80 learners of the foundation phase in sixteen schools was based on a survey study. The principle guiding the selection of this sample size was based on Strydom's (2005, p. 55) view that a 10% sample may be large enough to control for sampling errors. The questionnaire development was primarily guided by the literature review and subsequently by themes such as (1) remedy to challenges of numerical cognition and (2) developing conceptual understanding and computational fluency by introducing problem-solving skills.

Data and analysis

Guided by the teachers and research team and using a task-based interview design, the participants were guided to complete their respective research tasks as described in the participants and instrumentation section. Through the process of check-coding (themes), research teams' initial coding structures were then compared and contrasted, leading to the recognition of similar, different, and missing constructs, and the research team was able to reach a consensus. The codes developed and employed in this study analysis include those regarding; counting, reading numbers containing more than one digit, copying the numbers, mathematical signs, ability to finish tasks, learning with different kinds of objects(colours), and understanding mathematical language. The development of this list of themes extends most of the extant literature and theoretical basis regarding learners' uses of mathematical concepts in the foundation phase via (1) remedy to challenges of numerical cognition and (2) developing conceptual understanding and computational fluency by introducing problem-solving skills as explained in sample selection section.

Results

Due to the main research question: The first section addressed the learner's responses, while the research team in parallel considered another section which addressed the teachers' reflections due to the fact that teachers assisted learners in responding to the interview tasks. Incongruent to the main research question and characterised in the data analysis section, the reflections of the teachers were based on; mathematical concepts in the foundation phase via (1) remedy to challenges of numerical cognition and (2) developing conceptual understanding and computational fluency by introducing problem-solving skills. As indicated in the method section: The reason for the inclusion of teachers'

reflection was because a questionnaire that was posed to the learners gave an idea of the perception learners had about the problems they were encountering. However, a comprehensive analysis is achieved by way of including the judgements of the teachers.

Learners responses

As noted in both the literature and theoretical sections, while there have been ongoing studies on related topics, the study is delineated from the previous research and concurrently contributes to the existing body of work by examining challenges associated with numerical cognition in the early years. With reference is a guide through cognitive learning theory, employed to examine the challenges associated with early numerical cognition. Such assessment was achieved via examining semantics and textual misunderstanding in early numerical problems as well as concepts. The problems identified in prior research and which led to the formulation of the question statements in the questionnaire are presented in Table 1. These are considered below: Mistakes when counting, reading numbers that contain more than one digit and problems when copying numbers, mathematical signs confusion, finishing of tasks, using different kinds of colours, and understanding mathematical language. The results were organised into two sections.

Table 1. Response and associated themes

Themes	Response type			
	agreed	strongly agreed	disagreed	strongly disagreed
I always make mistakes when counting	50 (63%)	18 (23%)	7(9%)	5 (7%)
I have a problem in reading the numbers which contain more than one digit	25 (31%)	32 (40%)	11 (14%)	12 (15%)
I have problems when copying the numbers	60 (75%),	15 (19%),	3 (4%),	2 (3%)
mathematical signs confuse me	52 (65%)	11 (14%)	8 (10%)	9 (11%)
I always finish my tasks	7 (9%)	22 (28%)	15 (19%)	36 (45%)
I learn best when using different kinds of colours	45 (56%)	30 (38%)	10 (13%)	5 (7%)
I have problems understanding mathematical language.	35 (43%)	5 (6%)	20 (25%)	20 (25%)

In response to Statement 1, which stated that learners always make mistakes when counting, 50 (63%) learners agreed, 18 (23%) learners strongly agreed, 7 (9%) learners disagreed, and 5 (7%) learners strongly disagreed that they make mistakes when counting. This means that most of the participating learners (86%) agreed that they made mistakes when counting, and using numbers.

In response to Statement 2, which stated that learners had a problem in reading numbers that contained more than one digit, a total of 25 (31%) learners agreed, 32 (40%) learners strongly agreed, 11 (14%) learners disagreed, and 12 (15%) learners strongly disagreed that they had a problem with

numbers that contained more than one digit. Therefore, most of the learners (71%) agreed that they had problems in reading the numbers that contained more than one digit.

The learners were expected to respond to Statement 3 whether they had a problem when copying numbers. About 60 (75%), 15 (19%), 3 (4%), and 2 (3%) learners agreed, strongly agreed, disagreed, and strongly disagreed, respectively that they had a problem copying the numbers. Most of the participating learners (94%) agreed that they had such a problem.

Learners were expected to state in Statement 4 whether they were confusing mathematical signs. About 52 (65%) learners agreed, 11 (14%) learners strongly agreed, 8 (10%) learners disagreed, and 9 (11%) learners strongly disagreed that they had a problem with mathematical signs. Thus, again most of the learners (79%) agreed that they confused mathematical signs. For example, they reversed the numbers so that a 12 becomes a 21; 31 becomes 13; 15 becomes 51; and so on.

In response to Statement 5, learners were expected to state whether they always finished their tasks. About 7 (9%) learners agreed, 22 (28%) learners strongly agreed, 15 (19%) learners disagreed, and 36 (45%) learners strongly disagreed that they always finish their tasks. This highlights that most learners (64%), of whom 45% strongly believed this to be true, normally did not finish their tasks.

In response to Statement 6, learners were expected to state whether they learn best using different kinds of colours. About 45 (56%) learners agreed, 30 (38%) learners strongly agreed, 10 (13%) learners disagreed, and 5 (7%) learners strongly disagreed. Most of the learners (94%) thus agreed that the use of different colours helped them in the learning process.

In Statement 7, learners were expected to state whether they had a problem with understanding mathematical language. About 35 (43%) learners agreed, 5 (6%) learners strongly agreed, 20 (25%) learners disagreed, and 20 (25%) learners strongly disagreed.

Overall, the results showed the severity of the problems experienced by these learners as reflected in Table 2.

Table 2: Key numerical concepts in early childhood numerical cognition and their severity

Severity	Numerical concept
A total of 86% agreed that they made mistakes	when counting and using numbers
A total of 71% agreed that they had	problems in reading the numbers that contain more than one digit
About 64% (45% strongly believed this to be true)	normally did not finish their tasks
A total of 94% agreed that they had a	problem copying the numbers
A total of 79% of learners	confused mathematical signs, for example, by reversing the numbers
About 94% agreed that	the use of different colours helped them in the learning process

As reflected in Table 2: It was interesting to note that despite the previously elicited problems learners were experiencing, only about half of these learners agreed that they needed help because they did not understand the mathematical language, whereby they were given symbols to write. They also experienced problems with word sums as they were unable to read or spell. However, 50%

stated that they understood mathematical language and did not need help regularly. Given spatial structures in early numerical problems and concepts, one conclusion can be drawn based on the frequency of mistakes when counting, the problem in reading numbers that contain more than one digit, problems associated when copying numbers, mathematical signs being confused, the time needed to finish mathematical tasks, manipulatives such as using different kinds of colours, and the challenges associated with understanding mathematical language as the major influencers. Regarding mathematical language, learners seemed to make mistakes and confuse the mathematical signs, but they tended to learn better when using different colours. However, they seldom finished their tasks within the given time, mainly because they had problems with reading the numbers that contained more than one digit, and furthermore, they found it difficult to copy numbers.

Teachers responses

From the teachers' perspectives, in response to the numerical cognition in the foundation phase. Several reasons were identified. For instance, teacher 1 cited that "*...the extent to which numerical error analysis is becoming difficult to manage and be understood in schools...*" is indeed becoming a difficulty, so far received little to no attention have been., Wherein there added that this far, no formal training has been given as a continuous professional development (CPD) component of their training as educators. Similarly, teacher 5 maintained that "*...extent to which concepts need to be changed to promote and sustain learners' interest in Mathematics...*" has not been the focus of their CPD. As such, matters related to promotion and steadily re-addressing errors tend to receive little to no attention. While teachers 1 and 2 lament on little to no CPD to redress ongoing challenges, teacher 6 "*...the main characteristics reflecting the teachers' knowledge should entail that learners' levels of understanding...*" Such characterisation could "*...contribute to an awareness of the process of learning Mathematics...*" maintained the teacher 6. In support, there was the need to address the knowledge of the mathematical concepts that learners struggle to grasp. The excerpts from the sampled teachers are important to respond to the currently experienced gaps in educational theory and the knowledge among teachers in response to the theoretical demands. Thus, the consensus is that arithmetic cognition is not a question of whether one needs to activate numerical magnitude representation. This is because evidence from both learners and teachers suggests that it was clear enough to solve calculation problems. This question is important, especially when dealing with and managing numerical error analysis and the extent to which concepts need to be changed to promote and sustain learners' interest in Mathematics. Nevertheless, other issues, as suggested by teacher 9, which "*...needed to be addressed were error patterns in computation as exemplified....*" The unanswered question led to the teacher 6 "*... placing a renewed emphasis on the curriculum and assessment renewal...*" However, this (*curriculum and assessment renewal*) still lacks clarity regarding the extent to which error patterns can be used to improve cognition.

Subsequently, and as reflected in the current research through the responses of the learners and teachers, the current research examined whether tasks such as number comparison could be used as educational measurements to characterise both large cohorts of learners and also provide a deeper understanding of the performance of individual learners.

Discussion

Several ideas have been established in response to the question of; what numerical concepts in numerical cognition remain a concern in the foundation phase as guided by cognitive learning theory.

Numerical concept severity when counting and using numbers.

For instance, a total of 86% agreed that they made mistakes when counting and using numbers. As reflected in the responses of learners in Table 2 regarding when counting and using numbers and consistent with cognitive learning theory, Scholar (2008) argued that by the time learners leave primary school, they will not have confidently grasped counting and numbers. Consistent with teacher

6's response, who invariably *placed renewed effort on the curriculum and assessment. It could be argued that* such a lack of standard of competency provides a poor platform for learners to engage with algebra and other aspects of the school Mathematics curriculum when learners reach secondary school. However, if most of the learners do not achieve this competency during the early primary school years, such as reflected in Table 2, wherein a total of 71% agreed that they had problems in reading the numbers that contain more than one digit, as well as about 64% (45% strongly believed this to be true) normally, did not finish their tasks, then such deficit grows wider during the latter part of primary school, and becomes a serious problem when learners reach high school. This is consistent with teacher 5's assertion that "*...concepts need to be changed to promote and sustain learners' interest in Mathematics....*" Even though not an exhaustive explanation, while adding to work Abadzi (2006) suggested that there are three principles interlinked with mastering foundation phase numeracy. The first principle is the progress made in acquiring the number concept. The second principle refers to the shift from concrete to abstract reasoning. The third principle addresses the move from counting to calculating. Building on the work by Abadzi (2006), Gray and Reeve (2016) suggested that a significant leap in numerical literacy and understanding can be achieved when a learner moves away from regarding numbers as reflecting numerosity to objects, which can be manipulated according to certain rules. This implies changing an abstract concept to a visual one, a consequence of the current study. For instance, in primary school year 1; learners start to formalise ideas of addition by being introduced to the idea of counting on. This will require them to realise that addition can be carried out in either order. Similarly, they will be able to carry out the subtraction by finding the difference between two numbers by counting up.

Numerical concept severity: problems in reading the numbers that contain more than one digit and normally did not finish their tasks

However, in contrast, the current study revealed that a total of 71% agreed that they had problems reading the numbers that contain more than one digit, while a total of 79% of learners confused mathematical signs, for example, by reversing the numbers. What is meant and inconsistent with Zuj et al. (2017) is that this reflects why about 64% (45% strongly believed this to be true) normally did not finish their tasks. Regarding the statement related to the problem of reading the numbers that contain more than one digit, most of the learners confirmed that they had a problem. This referred to problems with the place value after interaction with some learners because for learners to succeed in differentiating the place of numbers (values), they must be able to know the place value, that is, hundreds, tens and units, and this is a crucial stage or level in mathematics. Thus, for the learners to be able to count, in addition, subtraction, multiplication, and division, they must know the place value.

Numerical concept Severity: problem copying the numbers and confusing mathematical signs, for example, by reversing the numbers

As reflected in Table 2, an inherent part of the study was to explore which errors (numerical concepts) exist when learners calculate and use numbers in mathematics in the foundation phase. Pipping (2005) suggested that errors of interference, in which different operations or concepts interfere with each other, will be of great value in exposing numerical errors. The majority of the learners agreed that they had problems when copying numbers, and this statement should inform teachers that they need to supervise learners to avoid them making many potential errors while doing their work in class, and even the parents need to monitor the learners' homework thoroughly. Several assessments could be made from [mis]understanding of the semantics of the mathematical text. For instance, besides Goswami's (2008) view, Buswell (1999, p. 19) cautioned that "errors in the learning of mathematics are simply the absence of correct answers or the result of unfortunate accidents." According to Buswell's (1999, p. 19) submission, misunderstanding of mathematical text is the consequence of specific processes whose nature could be discovered. Another form of error is one of assimilation, in which incorrect hearing causes mistakes in counting. Such errors result from a lack of attention and concentration (random or careless errors) (Yuan et al., 2016; Zuj et al., 2017). Lastly, the error of negative transfer from a previous task may occur, in which one could identify the effect of an erroneous impression obtained from a set of exercises. For instance, Flavell (1999) suggested that

learners who have been diagnosed as hyperkinetic are unable to focus or do cognitive tasks. Many years later, findings are still mixed on conceptual mathematical development (Cranfield, 2006; Pillay, 2010; Pournara et al., 2016; Schroder, Stewart, et al., 2014).

Numerical concept severity: the use of different colours helped them in the learning process

Considering that about 94% agreed that the use of different colours helped them in the learning process and consistent with Bornman's (2010) findings, particular difficulties are seen when learners have to read numbers that contain more than one-digit numbers and numbers that contain a zero, for example, 1007 or 1087. Furthermore, they may be confused when reading some numbers and reverse numbers; for example, where a 12 may become a 21, although, at other times, they might experience no difficulty with this. Thus, as recommended by Bornman (2010), the importance of the use of different colours thus helps learners in the learning process.

The summary thus far is that errors of interference arise when a previously learned skill or algorithm, because of its similar processes, makes the learning of a new skill or algorithm difficult. Additionally, having opportunities to make errors gives an opportunity for new ideas to be learnt by learners having to simulate or attempt hands-on problems, which gives learners the opportunity to make mistakes and then learn from them. Regarding the problems experienced by learners when copying numbers, this fact informs teachers that they need to supervise learners to avoid any such errors. Taking into account error analysis in the foundation phase of mathematics and thus a misunderstanding of the mathematical text, spatial structures in early numerical problems and concepts, various assessments have been made (Cranfield, 2006; Fleish, 2008; Gate, 2011; Halberda, 2008; Pausigere, 2011; Richards, 2005). Drawn from the study results as well as the earlier work by Richards (2005), for instance, error analysis in the foundation phase may be carried out as an aid for teaching to find out how well the learners learned and how a learner learns to count mathematically. Contrary to the insights gained from Pausigere (2011, p. 2), the suggestion is that "for mathematics classroom activities, teachers can let learners choose any number and allow them to investigate various ways of representing that number."

CONCLUSION AND IMPLICATION

In conclusion, understanding numerical cognition, while crucial, needs to account for various concepts that lead to such errors. The implication from the current study is that concepts such as counting and using numbers, reading the numbers that contain more than one digit, problem copying the numbers, confusion associated mathematical signs, finishing tasks and the use of different colours helped them in the learning process in teaching methods can be powerful. This is because the simultaneous application of different sources can create a better learning experience and help to minimise some of the identified problems in a more realistic manner for young learners. Another implication was that combinations of methods such as discussion, co-operative learning presentation, demonstration discovery, drilling, problem-solving, simulation, using tutorials and games, could also result in the educator reaching more learners in the classroom in an effective manner. The variation brought about by applying mixed teaching methods can thus support foundation learners to achieve the learning objectives

Further Research

Based on the data and the small sample size, claiming that the views shared by the participating teachers where a representative notion would unmistakably fall short of certainty and cast further doubt. Thus, it is essential to note that the objective of this study was not to generalise, but to provide an indication of the extent to which numerical cognition becomes more challenging to manage to promote and sustain mathematical interest in learners. Accordingly, the study also highlights the need for further research with a robust inferential standpoint as compared to the qualitative approaches that dominate the literature thus far.

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